Modality: Patterns, Theories, Logic I

Diti Bhadra
University of Minnesota

bhadra@umn.edu
Semantics Workshop at JNU, 09/2020

Material partially based on Portner (2009), Chapters 1 & 2.

1 Introduction

- ✤ *Modality* allows us the capacity to talk about objects and events that are displaced in space and time, as well as in actuality and potentiality.

- ✤ Given one of the design features of language is *displacement* (Hockett 1960), modal expressions are all pervasive in grammars.

- ✤ Modal auxiliaries and verbs: *can, could, should, must, would, need to, have to*
  Modal adverbs: *possibly, necessarily, maybe, probably, certainly*
  Modal adjectives, nouns: *possible, necessary, probable, certain, need, necessity, possibility*
  Propositional attitude verbs: *think, believe, hope, know, pleased*
  Generics, habituals, individual-level predicates: *A linguist studies languages, Diti teaches semantics, Ayesha is hilarious*
  Conditionals: *If Utpal writes a book on NPIs, then...*
  Tense and aspect: *I will go to the store, We are listening to a talk, We have taken a course on modality*
  Evidentiality: *It seems like there's a tiger in that shed*
  Covert modality with infinitives: *Tim knows how to solve the problem* (Bhatt 2008: "Tim knows how he *can* solve the problem.")

- ✤ Let's look at some examples from Hindi:

(1) a. *Abhi woh ghar pe* **hoga**. $hoga_{epistemic}$
         'Now he must be home.'

     b. *Abhi usse ghar pe hona* **chahiye**. $chahiye_{deontic/epistemic}$
         'He should be home now.'

     c. *Woh 10 minut mein 2 km daur* **sakta** *hain*. $sakta_{ability/deontic}$
         'He is able to/he may run 2kms in 10 minutes.'

d. *Mujhe 10 minut mein 2 km daur**na hai**.*

$\text{V-INF } hain_{(covert)deontic/bouletic/teleological}$

'I am required to/want to/have to run 2kms in 10 minutes.'

e. *Tum kehte toh woh kar **leta**.* $\text{V-}ta_{epistemic/counterfactual}$

'If you would have said, he would have done it.'

f. *Tumhe yeh kaam karna **padhega**.* $\text{V-INF } padhega_{deontic/teleological}$

'You have to/need to do this work.'

(2) *Ram yeh kar **sakta** hain.*

    a. Ram can do it, i.e. he is physically able to.

    b. Ram can do it, i.e. now he is allowed to/I am giving him permission to.

    c. Ram may/might do it, i.e. I am not sure if he certainly will but there is a possibility given what I know about him.

    d. Ram may do it, i.e. the moral ethics we live by do not prevent him from doing this.

    e. Ram can do this to achieve a particular goal.

➥ What we are already noticing is massive amounts of ambiguity/underspecification in *modal flavor*.

    ☞ Then should we consider the existence of these patterns as a result of accidental polysemy?

    ☞ Kratzer (1981, 1991): No! This is the result of **contextual dependency**.
    ☞ Modals by themselves have a skeletal meaning denotation; together with essential components from the context, a modal gets the particular flavor that it has in that particular context.
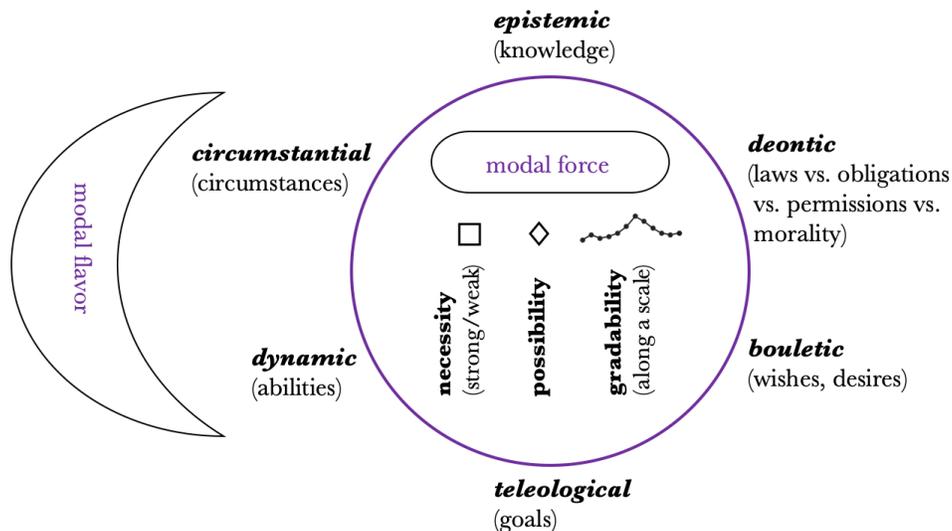

Figure 1: Modal *force* vs. modal *flavor*

- Any theory of modality in natural language has to explain how to account for these different flavors within the same framework, and correlate them with differences in modal force.

- We are going to look at two influential schools of thought bearing on this issue: modal logic (accessibility relations in particular), and natural language semantics (possible worlds in particular).

- There are many many aspects and facets of modality we will not be able to cover in this workshop: their interaction with scopal operators like *negation, quantifiers, conditionals, questions*, etc, their direct interaction with tense and aspect, their interaction with disjunction (*free choice*), clause-type related phenomena like *modal subordination*, relationship with *imperatives*, the very related phenomena of *evidentiality*, etc.

2 Accessibility Relations

- *Logic* comprises studying systems of reasoning; *Modal Logic* focuses on reasoning involving the concepts of necessity, possibility, implication, obligation, etc. (Portner 2009).

- Modal logic is not the same thing as the linguistics of modal expressions!

    - Modal logic aims to capture patterns of reasoning irrespective of natural language words like *must, possible, sakna, hona, zaroori, dorkar, uchit, uchitam*, etc.

    - Once you start studying the formal properties of operators like $\square$ and $\lozenge$, it can take you far far beyond the semantics of human language.

- In a modal logic language (MLL; Portner 2009), we first begin with some essential tools:

    (3)   Included in such a MLL are:
        a.   Infinite number of propositional variables: $p, q, r, s, t \ldots$
        b.   Negation: if α is a sentence in MLL, then so is α.
        c.   Conjunction, disjunction, conditionals: if α and β are sentences of MLL, then so are $\alpha \wedge \beta, \alpha \_ \beta, \alpha \,\tilde{N}\, \beta$.
        d.   Necessity and Possibility: if α is a sentence of MLL, then so are $\square\alpha$ and $\lozenge\alpha$.

- Note that some of these are *derivable* from other, more basic operators:

    (4)   a. $\alpha \wedge \beta = \mathbf{p}\, \alpha \_ \beta$
        b. $\alpha \,\tilde{N}\, \beta = (\, \alpha \_ \beta\mathbf{q}$
        c. $\square\beta = \lozenge\, \beta$

- Next, we come to two concepts that will get the "modal" part of things rolling (Hughes and Cresswell 1996):

3

(5)   FRAMES
A frame F is a pair $\langle W, R \rangle$ where W is (usually) a set of possible worlds, and R is a relation on W.

✦ This relation R is ultra important because it provides the mechanism for the worlds to "talk" to each other. For example:

(6)   a.   **Reflexive frame**: $\langle W, R \rangle$ is a reflexive frame iff for every $w \in W$, R(w, w).

b.   **Symmetrical frame**: $\langle W, R \rangle$ is a symmetrical frame iff for every w and $u \in W$, if R(w, u), then R(u, w).

c.   **Serial frame**: $\langle W, R \rangle$ is a serial frame iff for every $w \in W$, there is a $u \in W$ such that R(w, u).

d.   **Transitive frame**: $\langle W, R \rangle$ is a transitive frame iff for every $w, u, v \in W$, if R(w, u), and R(u, v), then R(w, v).

e.   **Equivalence frame**: $\langle W, R \rangle$ is an equivalence frame iff it is a reflexive, symmetrical, and transitive frame.

✦ Think of these frames as ways a world can talk to/access its friends that are far away from it.

↪ Different types of R thus gives us different types of ***accessibility relations*** (cf. Kripke semantics – a system created by Saul Kripke and Andre Joyal in late 1950s).
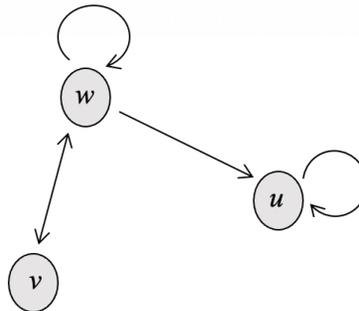

Figure 2: Accessibility between worlds

✦ Can we figure out the different relations/frames that exist between the worlds w, u, v?

✦ With the concept of a FRAME in place, we can define a MODEL:

(7)   MODEL
A model M is a pair $\langle F, V \rangle$, where F is a frame and V is a valuation function that takes a proposition in a $w \in W$ and returns a value of 1 or 0.

- For example, $V(w, p) = 1$, $V(u, q) = 0$, etc.

- We can do this for all atomic propositions as well all derived complex propositions containing operators like $\wedge, \_ , \tilde{N}, \Box, \Diamond$.

- We also have the useful logical property of ***validity*** in modal logic:

  (8) a. A sentence α is VALID in a MODEL $M = \langle W, R \rangle, V \rangle$ iff $⟦\alpha⟧^{w,M} = 1$ for $@w \in W$.

  b. A sentence α is VALID on a FRAME F iff, for every valuation function V, α is valid in the model $M = \langle F, V \rangle$.

- Thus, validity in a model is separate from validity on a frame (changing the valuation function changes the latter, for instance).[1]

- Take a reflexive frame, and if a sentence is valid on such a frame then it is T-VALID:

  (9) A T-VALID sentence: $\Box p \, \tilde{N} \, p$
  Natural language counterparts:
  a. If Ram must be at home now, then he is at home now.
  (where $must_{epistemic}$)

  b. If Sita must eat 10 rasgullas to win, then she will eat 10 rasgullas.
  (where $must_{deontic}$)

- Only (9-a) is T-VALID because the sentence is true, while (9-b) is not T-VALID because the sentence is not true.

  ↬ This tells us that T-validity can be useful for epistemic logic, but not for deontic logic.

- Take a serial frame, and if a sentence is valid on such a frame then it is D-VALID:

  (10) A T-VALID and D-VALID sentence: $\Box p \, \tilde{N} \, \Diamond p$
  Natural language counterparts:
  a. If Ram must be at home now, then he might be at home now.
  (where $must_{epistemic}$, $might_{epistemic}$)

  b. If Sita must eat 10 rasgullas to win, then she may eat 10 rasgullas.
  (where $must_{deontic-obligation}$, $may_{deontic-permission}$)

- With the flavors and subflavors as indicated, the sentence with deontic modals in (10-b) being D-VALID suggests that D-validity can be a property of deontic logic.

---

[1]There can be different kinds of validity based on the specific types of frame: K-VALID, T-VALID, B-VALID, D-VALID, S4-VALID, S5-VALID (Lewis 1918, Gödel 1933, Blackburn et al. 2002).

- ☞ Note that the sentence with the epistemic modals are thus both T-valid and D-valid (more on this on page 6 below).

- ❖ Our main stars for today, $\square$ and $\Diamond$, impose a **layer of quantification** on the accessibility relations between worlds:

(11) $⟦\square p⟧^{w,M} = 1$ iff for **every world** in W accessible to w, p holds (/is true) in all of those worlds.

- ☞ Crucially note that we have not said *what type* of accessibility relation holds between the worlds – this could be any type!
- ☞ So what is the $\square$ giving us – just the *quantity* of accessible worlds where the proposition in its scope holds.
  This is thus a natural way to think about **modal force**.

(12) $⟦\Diamond p⟧^{w,M} = 1$ iff for **some world** in W accessible to w, p holds (/is true) in that world.

- ❖ Now it's time to wonder: where does **modal flavor** then come from in this framework of modal logic?
  ☞ From the refined definitions of the accessibility relation R in terms of knowledge, rules, permissions, obligations, goals, desires, circumstances, abilities, etc.

- ❖ Thus, our frames and accessibility relations can now have special designations: (Note: I am using Portner's style of descriptions below (cf. Portner 2009) but most styles are sons and daughters of the formalizations in Kripke semantics; cf. also Hughes and Cresswell 1996, Blackburn et al. 2002).

(13)   **Epistemic frame**
  $F = ⟨W, R⟩$ is an epistemic frame iff for some individual i:
  a. $W =$ the set of possible worlds conceivable by humans.[2]
  b. $R =$ the relation which holds between two worlds w and u iff everthing which i knows in w is also true in u.

- ❖ This R inside this EPISTEMIC FRAME can be called an EPISTEMIC ACCESSIBILITY RELATION.

- ❖ What about the different properties of frames we learnt in (6)?
  ☞ They can be applied to each of these relations.

- ❖ For example, *knowledge* is considered to have the property of reflexivity, because if an agent *knows* a proposition in w, then it is true/is a fact in w.
  – thus, we can say that the epistemic frame is reflexive.

---

[2]Leibniz' work on modal metaphysics takes a possible world to a complete way that the universe could be throught its history; possible worlds accessible from our world could have minimal details changed (like maybe today is a Tuesday) or be completely different (maybe dinosaurs are still alive). Linguists ignore inconceivable worlds.

❖ Actually, if we assume an idealized account of knowledge where agents are perfect reasoners with infinite memory, then we can say that the epistemic frame actually contains an *equivalence* relation.
– (for example, recall that sentences with epistemic modals were both T-VALID and D-VALID to start with. cf. (10))
– but we know that that knowledge derived from different types of evidence and inferences sometimes admit errors, etc. (von Fintel and Gillies, 2010; Mihoc, Bhadra, and Falaus, 2019)

❖ With similar tools, we can define a DEONTIC FRAME and a corresponding DEONTIC ACCESSIBILITY RELATION inside it (assuming we are in the realm of rules for instance):

(14) **Deontic frame**
$F = \langle W, R \rangle$ is an deontic frame iff for some system of rules r:

a. $W =$ the set of possible worlds conceivable by humans.

b. $R =$ the relation which holds between two worlds w and u iff all of the rules which are established by r (the relation that associates each world in W with a set of rules) in w are followed in u.

❖ Again, thinking about properties of this frame, deontic frames are *serial*:
– by invoking seriality we obliterate the possibility of having an inconsistent set of rules
– because for every rule, there is a world in which the rule is followed (because the property of seriality requires that there be a corresponding world for every world accessible by the deontic accessibility relation).

❖ A disclaimer again: this is an idealized notion of a deontic relation, because we might find ourselves with contradictory requirements in reality.

❖ Now, with these tools in place, how do we apply them to our modal expressions?

(15) a. Necessity modals ($\square$): *must, should, would, zaroori*, V-INF *chahiye*, V-INF *padhega, nischoi, dorkar, uchit, uchitam*, etc.

b. Possibility modals ($\lozenge$): *may, might, can, could, sakna, shayad, hoyto, bodhoy*, etc.

❖ The central insight: for each modal flavor and *subtypes* of each flavor, there has to be a separate accessibility relation.

❖ Formally:

(16) $R_{epis}(w) = \{w' | w'$ is a world in which all the known facts in w hold$\}u$

(17) $R_{deontic}(w) = \{w' | w'$ is a world in which all the rules in w are followed$\}u$

❖ Within deontic necessity, there is a distinction between **strong** and **weak** necessity (*must, necessary, have to*, etc. vs. *should, ought*).

7

- Test 1 for detecting strength: strong necessity modals can reinforce weak ones but not vice-versa (von Fintel and Iatridou 2008):

  (18) a. You should wash your hands, in fact you must.

  b. ??You must wash your hands, in fact you should.

- Test 2 for detecting strength: Weak necessity modals are compatible with the negation of strong ones, but not vice versa (von Fintel and Iatridou 2008):

  (19) a. You ought to/should do the dishes, but you don't need to/have to.

  b. ??You need to/have to do the dishes, but it's not the case that you should/ought to.

- For an analysis of differential strength in deontic necessity in Bangla (with connections to Telugu and Hindi), see Bhadra and Banerjee (2020).

- Within modal logic, can we capture this relationship?: *must p |ù should p*

- Let r_{must} be the set of rules on which the accessibility relation of *must* – $R_{must}(w)$ is based on; and let r_{should} be the set of rules on which the accessibility relation of *should* – $R_{should}(w)$ is based:

  (20) $R_{must}(w) = \mathfrak{t}w'|@w's.t.R\mathfrak{p}w, w'\mathfrak{q}, V\mathfrak{p}p\mathfrak{q}\mathfrak{p}w'\mathfrak{q}$ 1u $^{\text{“}} r_{must}$

  (21) $R_{should}(w) = \mathfrak{t}w'|@w's.t.R\mathfrak{p}w, w'\mathfrak{q}, V\mathfrak{p}p\mathfrak{q}\mathfrak{p}w'\mathfrak{q}$ 1u $^{\text{“}} r_{should}$

- Both are deontic necessity modals ($\square$), thus crucially there is no difference in the denotations of the modals;
  – the main action (/difference) is in the sets of rules behind the accessibility relations.
  (Keep this insight in mind – it will carry over to the other school of thought we will be exploring tomorrow).
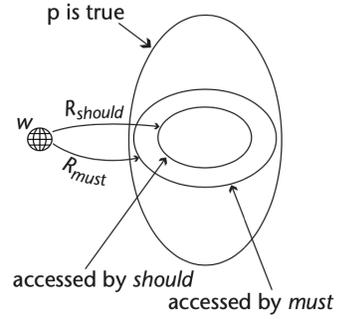
- Steps (let's name r_{must} as M and r_{should} as S):

  (22) a. if M „ S

  b. then $R_{should}(w)$ „ $R_{must}(w)$

  c. i.e. S „ M

  d. i.e. the subset relation reverses once we take intersections

- Now look at Portner (2009)'s representations of the relationship:
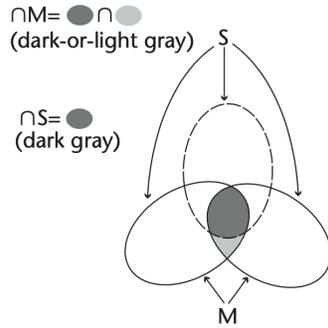
Intersection reverses the subset relation

Deontic *must p* entails deontic *should p*

- In the first figure: M contains less rules thus only the two solid ovals, while S contains more rules thus all three ovals.

- Thus, "*M* is larger than the "*S*, confirming the subset relationship in (22-c).

- **Insight:** the more things you care about, the smaller the intersection of worlds will be where all of those things are true.

- Thus, we can confirm *must p* |ù *should p*.
  – provided that there are at least two different kinds of deontic accessibility relations
  – figure on the right

- But deontic modality can have many other subtypes, and thus even these two deontic accessibility relations are not enough!
  – apart from the two kinds within $R_{deontic-rules}$, there has to be $R_{deontic-permissions}$, $R_{deontic-obligations}$, $R_{deontic-morals}$, etc.

  (23)   Just splitting apart deontic *must*:
    a.   [In view of the laws of Minnesota], drivers must yield to pedestrians.
    b.   [In view of the traditions of our family], you, as the youngest child, must touch the feet of all elders.
    c.   [In view of the rules of student-teacher relationships], you must not yell at your teachers.

- Kratzer (1977) points out that the kind of restrictions in [ ]-s that determine differences in accessibility relations can be infinitely many.

- **So what is starting to look like a problem here?**
  ☞ Masssive over-generation!

- Additionally, not to forget that there are several other flavors outside of deontic modality!:

  (24)   a.   Dogs **can** swim. (ability/dynamic)

    b.   Given how much you love semantics, you **should** attend the modality workshop. (<u>desire</u>)

    c.   To get into JNU, you **have to** study hard. (<u>goal</u>)

    d.   A pandemic **may** eventually wipe out large sections of humanity. (<u>history</u>)

(25) $R_{bouletic}(w) = \{w' | w'$ is a world in which all the desires of an agent i in w are satisfied$\}$

➤ The massive ambiguity/underspecification problem that we started out with is exacerbated by the possibility of the generation of multiple accessibility relations for the same modal:

(26)   *Ram yeh kar **sakta** hain.*

a.	$R(w, v)$, where $R_{dynamic}$	$\{w_1, w_2, w_3, w_4, \ldots\}$
b.	$R(w, v)$, where $R_{deontic-permissions}$	$\{w_5, w_6, w_7, w_8, \ldots\}$
c.	$R(w, v)$, where $R_{epistemic}$	$\{w_9, w_{10}, w_{11}, w_{12}, \ldots\}$
d.	$R(w, v)$, where $R_{deontic-morality}$	$\{w_{13}, w_{14}, w_{15}, w_{16}, \ldots\}$
e.	$R(w, v)$, where $R_{teleological}$	$\{w_{17}, w_{18}, w_{19}, w_{20}, \ldots\}$

➤ And then within modal logic, what prevents 10 other accessibility relations from being generated for *sakna*?[3]

➤ Thus, we understand the nature of the formal relations and properties better but have we made real progress towards providing an account of **modality in natural language**?

➤ 🙍: Unfortunately, no! Because our current logical system predicts many many more meanings for modals than they actually have.

➤ Enter: the field-changing work of Angelika Kratzer.
(tomorrow's journey)

---

[3]We have not even discussed the *indexicality* of modals (relativization to contextual parameters like speakers, hearers, times, locations); imagine how much more over-generation there can be once these factors are wired into the semantics.

References

Bhadra, D. and A. Banerjee (2020). What gerunds tell us about deontic necessity: A Relativized Ranked Priority Clusters model. Talk presented at *Sinn und Bedeutung 25* (`https://osf.io/yzbqh/`).

Bhatt, R. (2008). *Covert modality in non-finite contexts*, Volume 8. Walter de Gruyter.

Blackburn, P., M. De Rijke, and Y. Venema (2002). *Modal logic*, Volume 53. Cambridge University Press.

Gödel, K. (1933). Eine interpretation des intuitionistischen aussagenkalkülus. *Ergebnisse eines mathematischen Kolloquiums 4*, 3440.

Hockett, C. (1960). The origin of speech. *Scientific American 203*(3), 88–97.

Hughes, G. E. and M. J. Cresswell (1996). *A new introduction to modal logic.* Psychology Press.

Kratzer, A. (1977). What mustand canmust and can mean. *Linguistics and philosophy 1*(3), 337–355.

Kratzer, A. (1981). The notional category of modality. In Eikmeyer, Hans-Jürgen and Rieser, Hannes (eds.). *Words, Worlds, and Contexts*, 38–74.

Kratzer, A. (1991). Modality. In von Stechow, A. and Wunderlich, D., (eds.), Semantics: An international handbook of contemporary research. pp. 639–50.

Lewis, C. I. (1918). *A survey of symbolic logic.* University of California Press.

Mihoc, T., D. Bhadra, and A. Falaus (2019). Epistemic modals, deduction, and factivity: new insights from the epistemic future. In *Semantics and Linguistic Theory*, Volume 29, pp. 351–370.

Portner, P. (2009). *Modality*, Volume 1. Oxford University Press.

von Fintel, K. and A. S. Gillies (2010). Must... stay... strong! *Natural language semantics 18*(4), 351–383.

von Fintel, K. and S. Iatridou (2008). How to say ought in foreign: The composition of weak necessity modals. In *Time and modality*, pp. 115–141. Springer.